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Sovereign Credit Risk in a Hidden Markov Regime-Switching Framework. Part 1: Methodology

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Abstract

Standard approaches to estimating credit default probability estimation have certain drawbacks, most importantly regarding the underestimation of the true default probability which remains an undesirable property in sovereign risk management. As an alternative, this research applies a discrete-time Markov-modulated model to default probability estimation and applies it to Merton's contingent claims approach, offering an attractive combination of possibly resolving the underestimation inherent in most standard structural models with a more conservative approach when predicting valuable information from a sovereign's economic balance sheet. The crucial advantage of the estimation is that it backs the hypothesis that a regime-switching framework that allows for structural shifts can substantially improve default risk estimators, and proves that the methodology can be tractably extended to a contingent claims approach. Moreover, there are likely practical situations with certain policy implications when the predictions of the model could be used to detect systematic sovereign risk.

Introduction

The recent credit crisis has had a significant impact on the value of sovereign debt, raising the awareness of investors and regulators concerning the appropriate methods for valuation and risk management of these instruments. Many existing credit risk models assume homogenous market conditions and incorporating changes in market regimes or the economic environment due to a credit event appear difficult. This motivates the research for the formulation of an appropriate valuation model of a sovereign in a regime-switching framework. Regime switching aims to capture sudden changes in the economic climate or in the political setting.

Academic literature has been thorough in improving structural credit risk model specifications based on Merton's default model while increasing its flexibility [Black and Cox (1976), Longstaff and Schwartz (1995)]. Various studies have shown that the default probabilities implied by the Merton-type contingent claims approach can be inconsistent with historically observed default rates and there are a host of models that attempt to compensate for this. In this research, the model discussed in Gray et al. (2007) which extends Merton's contingent claims approach to the macro level to include a sovereign balance sheet analysis, is adapted to allow for structural changes and regime-switching with the intention of improving the default risk estimators. Specifically, we adapt a Markov-modulated version of the Merton structural model [Liew and Siu (2010)] for the valuation of the term structure of default probabilities (PDs) extracted from the market quotes of sovereign credit default swaps (CDSs) to predict a set of observed economic balance sheet information including a sovereign's asset value (A) and local-currency liability in foreign currency term (LCL). The economic intuition behind the regime-switching Markov-modulated model is to incorporate the impact of the macroeconomic conditions on the sovereign balance sheet information by assuming that the volatility of a sovereign's asset value has switching dynamics; they follow the evolution of a finite-state discrete Markov-chain where the states of the chain could represent the states of the economy. In this way, the research tries to establish a link between the credit market and a sovereign's balance sheet and attempts to understand if the credit market conveys useful information to predict economic balance sheet information.

Since September 2008, sovereign credit markets around the world have experienced an explosive growth in the trading activities of CDSs. In a CDS transaction, the protection buyer pays a series of fixed periodic payments (premium) to the seller in exchange for a contingent payment in case of a credit event [Duffie (1999)]. A credit event may include bankruptcy, a credit downgrade, or a failure to make the scheduled promised payments. This way, a CDS contract allows for the easy transfer of risk from the entity experiencing the credit event (the protection buyer) to the protection seller. Given the nature of the instrument, it is often identified as being the cause for the exacerbation of the European debt crisis, attracting much interest in policy circles and regulatory issues, including the ban of naked sovereign CDSs in the Eurozone. Until recently, perceived sovereign default risk in developed economies did not exist and liquidity in sovereign CDS spreads was concentrated in emerging markets. Following a tidal wave of incidents such as the Lehman Brothers bailout (September 2008), the downgrade of Greece (April 2010), and Ireland's (April 2011) debt to junk status, this perception was challenged as CDS premiums on sovereign bonds widened and sovereign solvency came under increased pressure.

The fact that CDSs do not require assumptions about the benchmark risk-free rate is regarded by some to be a clearer indicator of the time-series and cross-sectional information on a sovereign's credit quality than a government bond [Duffie et al. (2007)]. In addition, there is some empirical evidence that the CDS market can influence the price of the government bond market when the market is distressed [Delatte et al. (2012)]. This suggests that CDSs could be used to establish and disseminate price information, leading to the following questions: does financial speculation with CDSs play a positive punitive role by forcing governments to adjust their fiscal policies? Or have government bond spreads merely become sensitive to speculative movements? This remains a contentious issue still under intense debate [Duffie (2010)]. Either way, the high costs associated with a credit event highlight the need for a comprehensive approach to assess vulnerabilities or the robustness of a country's financial system and economic health. Such an approach would require a method of analyzing a sovereign's balance sheet to evaluate and measure credit and market risks used for forecasting credit spreads.

Balance sheet risk is considered to be key to understanding the credit risk and crisis probabilities of any entity, in this case a sovereign. Uncertainty in sovereigns' future asset values directly translate into default risks while price, flow, and liquidity shocks eventually convert to credit risk in a crisis. Financial fragility is thus very closely connected to default probability. The risk of default therefore occupies a central role in the measurement and hedging of sovereign credit risk.

Research focus

Motivated by the growth in sovereign CDS trading activity, the increased pressure on the solvency of sovereigns under changing economic conditions and the sparse literature on sovereign credit risk, this research proposes a comprehensive approach to measure, analyze, and model sovereign credit risk in a way that can help forecast sovereign default rates and evaluate the impact on hedging strategies. Specifically, this research addresses factors which affect non-linearity in default probabilities such as structural breaks in the stochastic processes governing valuations and pertinent economic events rarely accounted for in sovereign credit risk.

Furthermore the research proposes the valuation of default probabilities under an extended version of Merton's structural model represented by a discrete-time hidden two-state Markov regime-switching Gaussian model, whose states represent the hidden states of the economy and distinguishes a strong economy from a weak economy or a normal market from one experiencing an economic crisis. The valuation of sovereign debt as an option thus incorporates the impact of structural changes in economic conditions. The model parameters include the volatility of a risky asset and the leverage, which is calibrated using market quotes of sovereign CDSs. We perform a sensitivity analysis of the model which highlights the effects of model parameters on the term structure of default probabilities. The regime-switching framework allows for a time-varying rate of default and captures the effect of the environment through patterns of persistence which could arise when structural shifts occur. This could allow for a more defensive tilt on the sovereign states replicating portfolio when a change in economic regime is imminent.

The estimates of the model parameters are used to calculate the price of a standard European call option according to the contingent claim approach (CCA). The observed local-currency liability (LCL) can be compared with the valuation implied in market quotations through the switching regime model. This allows us to understand how much CDSs, conditional to the use of the proposed model, can tell us about the market estimate of the value of sovereign assets. In practice, there exist significant differences between the two. In a follow up paper, a detailed empirical analysis will be performed on a set of countries, representative of advanced and emerging economies.

This research aims to contribute to the rapidly growing literature on the application of structural credit models to sovereign debt in an attempt to understand the effect of an economic regime shift on valuations and its link to the sovereign balance sheet. The research has two main contributions. First, a readily implementable approach to modeling sovereign default is presented by adapting a Markov regime-switching technique. This framework, rich but tractable, allows us to capture very different shapes in the term structure of PDs and credit spreads, which it is not the case in the Gaussian structural approach. Second, the research applies the CCA model to measure sovereign debt providing insights into how the proposed approach can be used to predict economic balance sheet information.

The paper is organized as follows. In the next section we give an overview of the use of structural models for measuring default risk and we provide the background to the use of regime-switching models in finance. We then present the general setup behind this class of models and how they may be used to estimate default probabilities (PD). A sensitivity analysis section follows, where we discuss the effect of the different parameters on the shape of the PD term structure. We conclude by discussing the model calibration and how the proposed approach is used to estimate the value of foreign sovereign assets. A concrete case is discussed with reference to Brazil. In a follow-up paper we will consider a more extensive empirical analysis with applications to different countries.

Review of credit risk modeling

Structural models of credit risk which follow the CCA have a long tradition in modern financial theory – see for example Sundaresan (2000) for an early survey on credit risk models. Pioneered by Merton (1973), its application stems from the analysis of corporate sector credit risk [for example, Black and Scholes (1973), Crouhy et al. (2000), Leland (2004)], the financial sector [for example, Merton (1977) and Kupiec (2002)] with a fast-growing focus at a sovereign level [for example, Gray et al. (2007), Gapen et al. (2008)].

The CCA serves to predict a relationship between credit spreads, leverage, asset volatility, and interest rates. The merits of the CCA, when analyzing sovereign risk and contributing to policy design and risk management, lie in the ability to provide a structural interpretation of a sovereign's balance sheet, and translate changing economic conditions directly into quantitative credit risk indicators [Gapen et al. (2008)]. There is, however, strong evidence that the Merton-style structural model underestimates the actual probability of default [Tarashev (2005), Leland (2004), Boral et al. (2000)]. The underestimation is attributed to the fact that a firm or entity's value is described by a diffusion process, which does not allow for sudden jumps in the entity's value. Hence default can never occur by surprise [Erlwein et al. (2008)]. Leland (2004) suggests a jump component as a possible way of improving the underestimation. This could be achieved by a jump-diffusion model or a Markov regime-switching model wherein the jumps or discontinuities in a sovereign's asset value occur at the instant where the transition of macroeconomic conditions takes place. Another possibility is to consider an uncertain threshold level. Such an approach generates default probabilities that can be expressed as mixtures and is a simplified case of what is obtainable with regime-switching models. In addition, it allows one to capture early default by introducing an ad hoc specification of the threshold default level. Numerical experiments confirm that this approach performed very well in predicting the Lehman default [Brigo et al. (2009)].

The structural approach to credit risk is based on three principles: i) the values of liabilities are derived from assets, ii) liabilities are separated into junior and senior claims according to priority, and iii) assets follow a stochastic process. Gray et al. (2007) suggests its extension to sovereign debt. The LCL in foreign currency terms is treated as a call option on a sovereign entity's assets just as a traditional corporate debt model would treat equity as a call option on debt. The foreign-currency liability is treated as a risk-free asset less a put option on the implied asset value. Currency liabilities, both foreign and local, are observable in the market while the sovereign entity's asset value can be obtained using the Black-Scholes and Merton formula for option pricing.

In this paper, motivated by the considerations such as structural changes and innovations in the legal, macroeconomic, regulatory, and capital-markets environment, we propose to model sovereign assets via regime-switching models, which surprisingly are not so frequently studied in the credit risk literature and in application to pricing sovereign debt. Regime-switching models are based on the natural idea that the economic environment is not stable but subject to regular changes at non-predictable stopping times; a simple illustration being the successive periods of expansion and recession in the economy or even a political change. These changes should induce in a model a sudden modification of the underlying parameters while the "all other things are held fixed" economic assumptions would break down.

The study of hidden Markov models started in the late 1960s by Baum and Petrie (1966) and has since been applied to various research fields. These models were introduced by Hamilton (1994) as regime-switching models widely used to detect turning points or structural breaks in econometrics. Timmerman (2000) studied the capability of the model to capture asset price moments while Buffington and Elliot (2002) used it for option pricing, exploiting the characteristic function and Fourier inversion method. Applications of regime switching models to financial derivatives, interest rates, and portfolio optimization have mainly been explored in a continuous time Markov switching framework. Drawbacks of optimizing in the continuous time include the memory-less property of this technique which some argue is inadequate when analyzing real world data.

In the discrete framework, Ishijima and Kihara (2005) develop a European call option pricing formula using the Baum-Welch algorithm. Giesecke et al. (2011) estimate the model via a genetic search algorithm that maximizes a log-likelihood function to forecast realized default rates. Liew and Sui (2010) derive an analytic European call option price based on the Ishijima and Kihara (2005) approach using a recursive formulation which makes the model implementation easy. Here we adopt their approach and use it for an analysis of sovereign debt.

The adopted Markov regime-switching model

This research adapts the estimation and option valuation to the case of a sovereign to extract key parameters in order to measure a sovereign's credit risk.

Consider a hidden Markov regime-switching Gaussian model for asset prices in a discrete-time economy. Let $\mathcal{T}$ be the indexed time set $\{0,1,2,\ldots,T\}$ where $T<\infty$ and represents the points at which an economic activity occurs. The evolution of the hidden state of the economy over time is described as follows:

Let $X:=\{X_t | t \in \mathcal{T}\}$ be a discrete-time finite state, hidden Markov chain on $(\Omega, \mathcal{F}, \mathbb{P})$. The state space of the Markov chain X is identified by a set of standard unit vectors $\mathcal{E}:=\{e_1,e_2,\ldots,e_N\}$ where $e_i = (0,\ldots,1,\ldots,0)' \in \Re^N$ so $\langle e_i,e_j\rangle = \delta_{i,j}$, the Kronecker delta. We assume that the Markov chain X is time-homogenous and the probability law of X is specified by its transition probabilities and initial distribution.

For each i,j = 1,2,…,N let $a_{ji} := \mathbb{P}(X_{t+1}= e_j \mid X_t = e_i)$. This is the probability of switching from state i at time t to state j in the following period. Write $\mathcal{A}$ for the transition probability matrix $[a_{ji}]_{i,j=1,2,\ldots,N}$ of the chain X under $\mathbb{P}$. Let $p:=(p_1,p_2,\ldots,p_N)' \in \Re^N$ where $p_i := \mathbb{P}(X_0 = e_i)$ so that p is the initial distribution of the chain X, which is assumed to be stationary.

To give the reader a better understanding of the behavior of the switching regime model we can consider Figures 1 and 2. We illustrate a simulated path of the log-leverage parameter $\ln\left(\frac{S_t}{K}\right)$, up to a one-year horizon in Figure 1. As a switch in regime to state two occurs (bottom panel), a sudden increase in the volatility is observed in the log-leverage trajectory. Figure 2 shows the simulated probability density function at a one-year horizon for the log-leverage parameter. The two states differ for the risk-free rate and the volatility; 1% and 10% in the good state and 9% and 50% in the bad state respectively. The probability of remaining in the good state is 0.98, whilst the probability of remaining in the bad state is 20%. The right panel shows the tails of the distribution that appear much fatter than the superimposed Gaussian with same unconditional mean and unconditional variance.

Liew and Siu (2010) derive an analytical formula for pricing a standard European call option with strike K and maturity at time T while assuming a two-state Markov chain (N=2). First quantities and measures relating to the hidden Markov chain X are introduced. For any i,j=1,2,…,N and for each $t \in \mathcal{T}\backslash\{0\}$, define:

$J^i(t,T) := \sum_{k=t}^{T-1} \langle X_k, e_i \rangle$: the occupation time of the chain X in state e_i up to T–1;

$N^{j,i} := \sum_{k=t}^{T-1} \langle X_k, e_i \rangle \langle X_k, e_j \rangle$: counts the number of transitions from state e_i to e_j up to time T.

Consider a standard European call option with strike price K and maturity T and suppose the current time is t. The option price formula in the case that N=2 is given by:

$$C(t,T) = S_t \sum_{k=0}^{T-t} \mathbb{P}(J^1(t,T) = k)\Phi\left(\frac{\log\left(\frac{S_t}{K}\right)+k\left(r_1+\frac{\sigma_1^2}{2}\right)+(T-t-k)k(r_2+\frac{\sigma_2^2}{2})}{\sqrt{k\sigma_1^2+(T-t-k)\sigma_2^2}}\right)$$
$$- K \sum_{k=0}^{T-t} \mathbb{P}(J^1(t,T) = k)e^{-kr_1-(T-t-k)r_2}\Phi\left(\frac{\log\left(\frac{S_t}{K}\right)+k\left(r_1-\frac{\sigma_1^2}{2}\right)+(T-t-k)k\left(r_2-\frac{\sigma_2^2}{2}\right)}{\sqrt{k\sigma_1^2+(T-t-k)\sigma_2^2}}\right) \quad (1)$$

Here $S:=\{S_t | t \in \mathcal{T}\}$ is the price process of the risky asset, K is the default threshold known as the strike, $\sigma:=(\sigma_1,\sigma_2)' \in \Re$ the volatility of the risky asset when the economy is in state i=1,2.

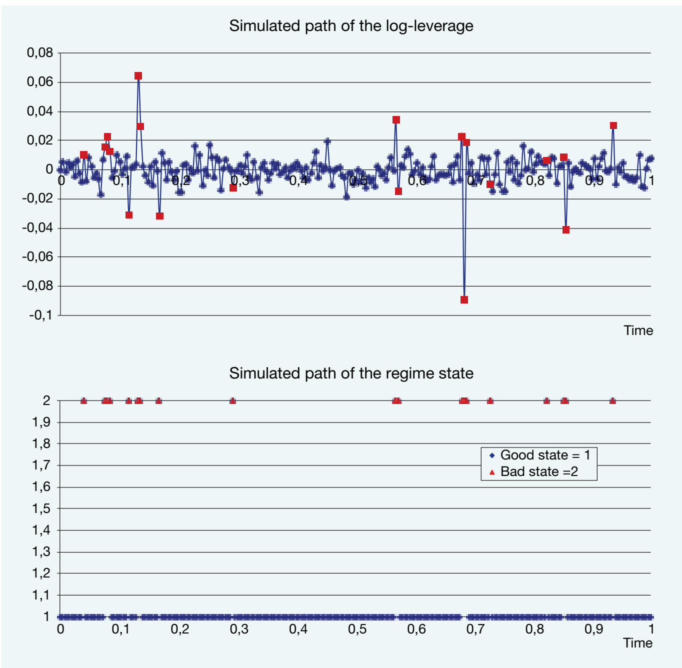

Figure 1 – A simulated path of the log-leverage parameter

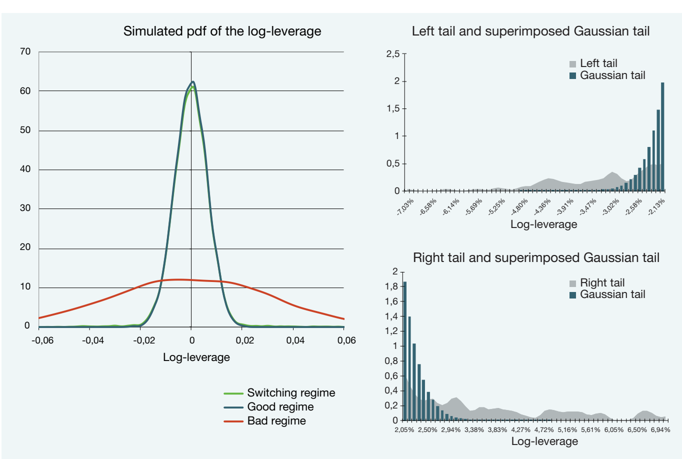

Figure 2 – Simulated density probability function at one-year horizon for the log-leverage parameter

The following theorem by Bhattacharya and Gupta (1980) provides an analytical formula for the probability function $\mathbb{P}(J^1(t,T) = k)$. In the market, the economic state is unobservable so when the state is hidden, a trajectory of transitions until maturity should be considered. Let $\{X_t\}_{t=0}^{\infty}$ be a stationary first-order Markov chain with state space $\mathcal{E}:=\{e_1,e_2\}$ and transition probability $a_{ji} = \mathbb{P}(X_{t+1} = e_j \mid X_t = e_i)$ for i,j=1,2 and $t \geq 1$ and assume that none of the transition probabilities is zero or one. Given $X_0 = e_i$ (the initial state of the chain being i), let $N^m_{i,j}$ be the number of e_j's in $\{X_1,X_2,\ldots,X_m\}$. Then

$$\mathbb{P}(N_{11}^m = k) = \begin{cases} (1-a_{11})a_{22}^{m-1} \text{ if } k=0 \\ a_{22}^{m-2k-1}(a_{12}a_{21})^k \sum_{j=0}^{k}\binom{k}{j}\left(\frac{a_{11}a_{22}}{a_{21}a_{12}}\right)^j \left[a_{22}\binom{m-k-1}{k-j-1} + a_{21}\binom{m-k-1}{k-j}\right] \text{ if } 0<k<m \\ a_{11}^m \text{ if } k=m \end{cases} \tag{2}$$

and

$$\mathbb{P}(N_{21}^m = k) = \begin{cases} a_{22}^m \text{ if } k=0 \\ a_{22}^{m-2k}a_{11}^{k-1}\,a_{12}^k \sum_{j=0}^{k-1}\binom{k-1}{j}\left(\frac{a_{11}a_{22}}{a_{21}a_{12}}\right)^j \left[a_{22}\binom{m-k}{k-j-1} + a_{21}\binom{m-k}{k-j}\right] \text{ if } 0<k<m \\ a_{12}a_{11}^{m-1} \text{ if } k=m \end{cases} \tag{3}$$

Rewriting in the notation for $1 \leq k \leq T - t - 1$

$$\mathbb{P}(J^1(t,T) = k \mid X_t = e_1) = \mathbb{P}\left(N_{11}^{T-t-1} = k-1\right) \tag{4}$$

$$\mathbb{P}(J^1(t,T) = k \mid X_t = e_2) = \mathbb{P}\left(N_{21}^{T-t-1} = k\right) \tag{5}$$

Therefore

$$\mathbb{P}(J^1(t,T) = k) = \begin{cases} a_{22}^{T-t-1}\langle \widehat{X}_t, e_2\rangle & \text{if } k=1 \\ \mathbb{P}\left(N_{11}^{T-t-1} = k-1\right)\langle \widehat{X}_t, e_1\rangle + \mathbb{P}\left(N_{21}^{T-t-1} = k\right)\langle \widehat{X}_t, e_2\rangle & \text{if } 1 \leq k \leq T-t-1 \end{cases} \tag{6}$$

With T being fixed, and replacing the transition probabilities with their estimates, it is possible to compute $\mathbb{P}(J^1(t,T) = k)$ for each k=1,2,…,T - t - 1.

The following section provides the key contribution of the proposed research, describing the process of calculating the probability of default on a N-state hidden Markov Model, and derives the value for a sovereign's assets and a local-currency liability in foreign currency terms. For the purpose of this research, a N = 2 state hidden Markov model is considered for ease of implementation while keeping the model parsimonious. In practice, a Markov chain with two states sufficiently distinguishes between stable market conditions and one experiencing a crisis. See for example the discussion in [Erlwein et al. (2008)].

Extracting sovereign riskiness from a Markov-modulated Merton model

In the Black-Scholes formula for European calls, $N(d_2)$ represents the risk-adjusted probability that the option will be exercised (S > K). In the credit Merton model, the Black-Scholes formula is relevant because it treats equity as a call option on an entity's assets. By translation, $N(d_2)$ becomes the probability that the entities asset value S will exceed the default threshold K at the end of time, so that $1-N(d_2)$ becomes the probability of default. In the Merton pricing formula, d_2 is known as the "distance to default." The firm leverage is the ratio between firm value and threshold (S/K). If at time T, the leverage falls below 1, the firm will default.

By a similar reasoning, in the Markov regime-switching, given the option price formulae in Equation (1), the probability of default can be defined as:

$$\boldsymbol{PD(t,T)} = \sum_{k=0}^{T-t} \mathbb{P}(J^1(t,T) = k)\boldsymbol{\Phi}\left(\frac{log\left(\frac{S_t}{K}\right)+k\left(r-\frac{\sigma_1{}^2}{2}\right)+(T-t-k)k\left(r-\frac{\sigma_2{}^2}{2}\right)}{\sqrt{k\sigma_1{}^2+(T-t-k)\sigma_2{}^2}}\right) \quad (7)$$

Notice that the above probability of default turns out to be a mixture of probabilities. A similar result in a much more simplified framework can be obtained by making the default threshold stochastic, as in Brigo et al. (2009). This probability of default depends on known parameters such as the interest rate r and the time to maturity T, and on additional parameters that need to be estimates, such as σ_i, $a_{i,j}$, S_t, K (or equivalently $\frac{S_t}{K}$) and X_0 for i,j = 1,…,N. In addition we have also to estimate the initial probability of being in state i i.e., p_i.

A sensitivity analysis of the default probability

Before illustrating the application to the estimation of sovereign debt, we perform a sensitivity analysis on the term structure of default probabilities. This allows us to appreciate how the effects of changing economic conditions can be captured by the Markov regime-switching model and how this is impounded in default probabilities. In addition, the model framework also avoids the problem of the underestimation of default probabilities associated with the Merton structural model [Erlwein et al. (2008)]. For ease of illustration we consider a two-state Markov chain, where states 1 and 2 represent a "stable" and "bad" economy respectively. This sufficiently distinguishes a strong economy from a weak economy or a normal market from one experiencing an economic crisis [Erlwein et al. (2008)].

First, numerical results are presented for varying values of a_{11} and for different choices of the leverage parameter $\frac{S}{K}$. In the first numerical part, a_{11} is set to 0.4, 0.5, 0.6, 0.7 and 0.8 with different values of the leverage parameters $\frac{S}{K}$ = 1.2, 1.6, 2.0, 2.4 and 2.8. Default occurs as soon as the leverage falls below 1.

Case I: The impact of a_{11}, S/K and T

Table 1 displays the default probability for varying values of the transitional probabilities a_{11} in the transition matrix and the leverage parameters $\frac{S}{K}$ for different maturities T= [1 2 3 4 5 7 10] years.

The regime-switching model allows for switching of the economy between "good" (we label it as state 1) and "bad" states (we label it as state 2). Therefore the lower a_{11}, i.e., the probability of remaining in the good state, the higher the risk of default and so the higher the default probability. This is illustrated in Table 1 comparing the default probabilities for different values across columns.

A similar effect is observable when the leverage parameter $\frac{S}{K}$ decreases from 2.8 to 1.2 in Table 1; the asset value is more likely to fall below the default barrier level. Therefore the probability of defaulting, ceteris paribus, will increase. Finally the default probability increases as the time to maturity increases; the longer the time to maturity, the greater the risk of default. In Figure 3 the sensitivity analysis of the term structure of default probabilities for varying values of the transition probabilities a_{11} are shown for a constant leverage $\frac{S}{K}$ = 1.2.

Figure 4 illustrates the default probabilities for varying values of S/K, holding a_{11} constant.

		Default probability				
Leverage	Maturity	a_{11} = 0.4	a_{11} = 0.5	a_{11} = 0.6	a_{11} = 0.7	a_{11} = 0.8
S/K = 1.2	T=1	0.13%	0.13%	0.13%	0.13%	0.13%
	T=2	3.67%	3.08%	2.49%	1.90%	1.60%
	T=3	5.30%	4.79%	4.13%	3.33%	2.87%
	T=4	6.66%	6.38%	5.85%	5.00%	4.44%
	T=5	7.50%	7.42%	7.03%	6.24%	5.65%
	T=7	7.98%	8.04%	7.78%	7.11%	6.55%
	T=10	8.34%	8.47%	8.32%	7.76%	7.24%
S/K = 1.6	T=1	0.13%	0.13%	0.13%	0.13%	0.13%
	T=2	2.99%	2.51%	2.04%	1.56%	1.32%
	T=3	4.44%	4.03%	3.49%	2.82%	2.44%
	T=4	5.94%	5.74%	5.31%	4.57%	4.07%
	T=5	6.93%	6.90%	6.58%	5.87%	5.33%
	T=7	7.52%	7.63%	7.42%	6.79%	6.26%
	T=10	7.97%	8.15%	8.03%	7.49%	7.00%
S/K = 2.0	T=1	0.13%	0.13%	0.13%	0.13%	0.13%
	T=2	2.46%	2.07%	1.68%	1.29%	1.10%
	T=3	3.76%	3.43%	2.99%	2.43%	2.10%
	T=4	5.36%	5.24%	4.88%	4.23%	3.78%
	T=5	6.47%	6.49%	6.23%	5.58%	5.07%
	T=7	7.15%	7.30%	7.13%	6.54%	6.04%
	T=10	7.66%	7.89%	7.79%	7.28%	6.81%
S/K = 2.4	T=1	0.13%	0.13%	0.13%	0.13%	0.13%
	T=2	2.05%	1.73%	1.41%	1.09%	0.93%
	T=3	3.22%	2.96%	2.59%	2.11%	1.83%
	T=4	4.90%	4.83%	4.53%	3.96%	3.54%
	T=5	6.09%	6.16%	5.94%	5.35%	4.87%
	T=7	6.84%	7.03%	6.90%	6.35%	5.87%
	T=10	7.41%	7.68%	7.61%	7.12%	6.66%
S/K = 2.8	T=1	0.13%	0.13%	0.13%	0.13%	0.13%
	T=2	1.75%	1.48%	1.21%	0.94%	0.80%
	T=3	2.82%	2.60%	2.29%	1.87%	1.63%
	T=4	4.55%	4.52%	4.28%	3.75%	3.36%
	T=5	5.80%	5.91%	5.73%	5.17%	4.72%
	T=7	6.60%	6.83%	6.73%	6.20%	5.74%
	T=10	7.21%	7.51%	7.47%	7.00%	6.55%

Table 1 – Default probabilities for varying levels of a_{11} and leverage parameter S/K

Case II: The impact of σ_1 and σ_2

Next, a sensitivity analysis for the default probability curve is performed when the volatilities σ_1 and σ_2 vary. For this sensitivity analysis, the transition probabilities a_{11} and a_{12} are set to a high value of 0.85 and 0.99 respectively, implying a persistence in state 1, whilst the leverage parameter is held constant at $\frac{S}{K}$ = 1.5.

Figure 5 illustrates the default probabilities calculated varying σ_1, setting the value of σ_2 to 0.5. On investigation, it becomes apparent that the default probabilities are sensitive to the volatility of the asset value and the default probabilities increase as σ_1 increases. A similar analysis, changing σ_2 while keeping constant the value for σ_1 = 0.1, is illustrated in Figure 6; default probabilities increase as σ_2 increases. Higher volatility increases the likelihood that the leverage will fall below 1. The numerical results related to this analysis are presented in Table 2.

Brigo and Tarenghi (2005) highlight that structural models often imply unrealistic short-term credit spreads. Figure 7 shows that in the regime-switching model, default probabilities for very short maturities are not zero. This motivates the use of this simple and tractable model for sovereign credit risk estimation. This will be illustrated in more detail in the next section.

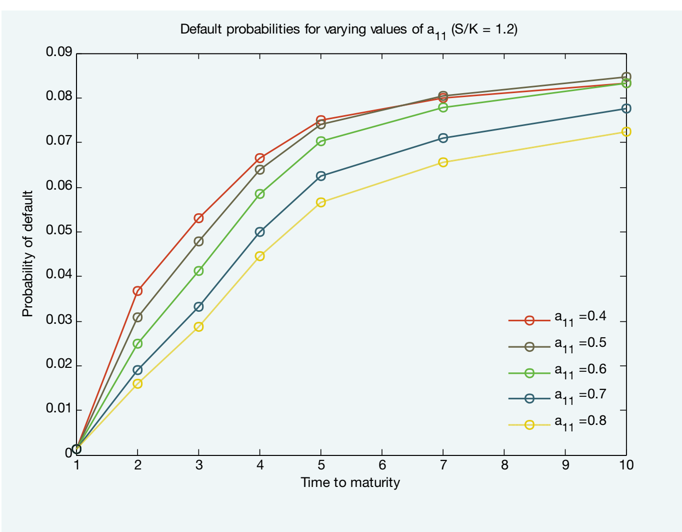

Figure 3 – Term structure of default probabilities for varying values of a_{11}

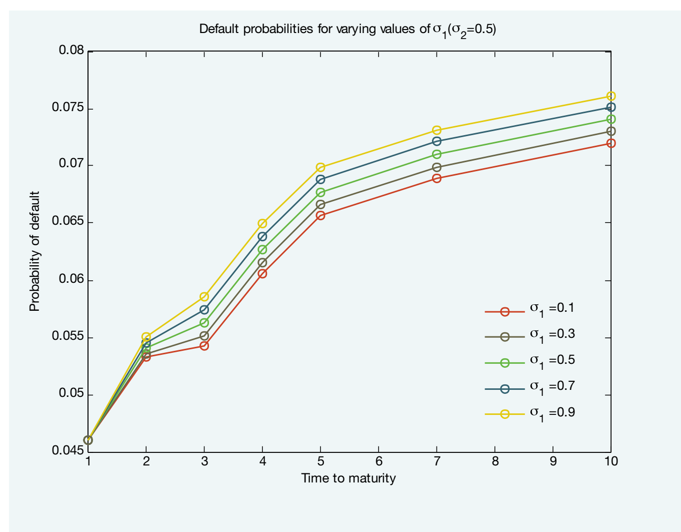

Figure 5 – Term structure of default probabilities for varying values of σ_1

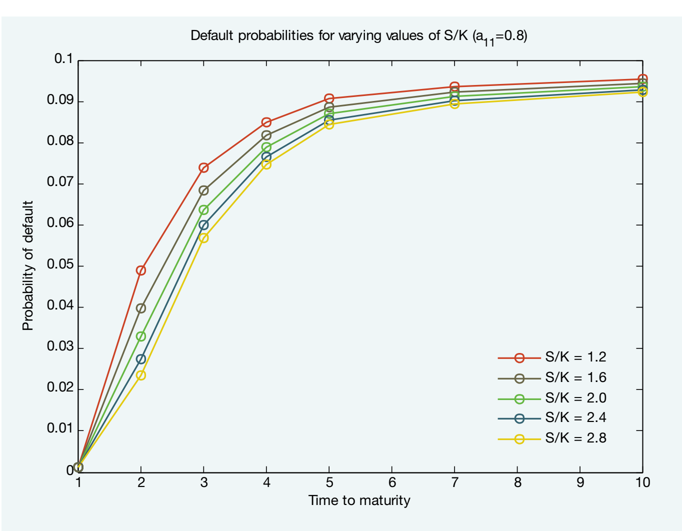

Figure 4 – Term structure of default probabilities for varying values of S/K

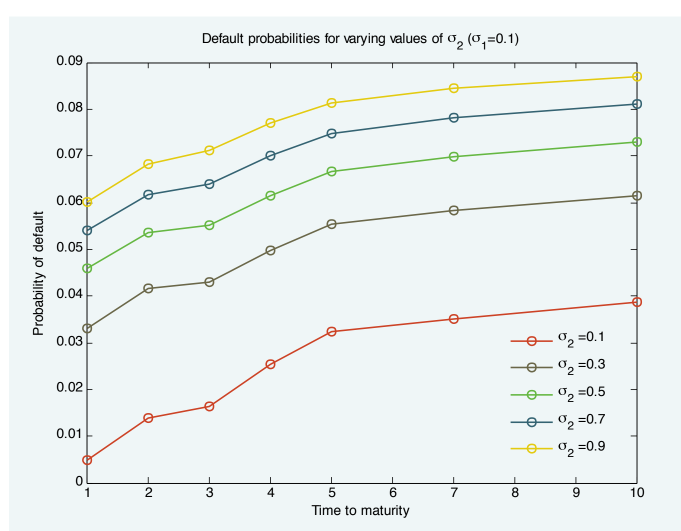

Figure 6 – Term structure of default probabilities for varying values of σ_2

State volatility	Maturity	σ1 = 0.1	σ1 = 0.3	σ1 = 0.5	σ1 = 0.7	σ1 = 0.9
				Default probabilities		
σ2 = 0.1	T=1	0.49%	0.49%	0.49%	0.49%	0.49%
	T=2	1.15%	1.38%	1.51%	1.59%	1.65%
	T=3	1.10%	1.64%	1.94%	2.15%	2.31%
	T=4	2.02%	2.54%	2.84%	3.05%	3.22%
	T=5	2.75%	3.23%	3.53%	3.75%	3.92%
	T=7	3.04%	3.52%	3.84%	4.07%	4.25%
	T=10	3.42%	3.88%	4.20%	4.45%	4.63%
σ2 = 0.3	T=1	3.30%	3.30%	3.30%	3.30%	3.30%
	T=2	4.09%	4.16%	4.24%	4.31%	4.36%
	T=3	4.11%	4.29%	4.47%	4.63%	4.77%
	T=4	4.80%	4.98%	5.16%	5.31%	5.45%
	T=5	5.36%	5.53%	5.70%	5.85%	5.99%
	T=7	5.65%	5.84%	6.00%	6.16%	6.30%
	T=10	5.97%	6.15%	6.32%	6.47%	6.60%
σ2 = 0.5	T=1	4.60%	4.60%	4.60%	4.60%	4.60%
	T=2	5.33%	5.36%	5.41%	5.46%	5.50%
	T=3	5.43%	5.51%	5.62%	5.74%	5.86%
	T=4	6.06%	6.15%	6.26%	6.38%	6.49%
	T=5	6.57%	6.66%	6.77%	6.88%	6.99%
	T=7	6.89%	6.99%	7.10%	7.21%	7.31%
	T=10	7.20%	7.30%	7.41%	7.51%	7.61%
σ2 = 0.7	T=1	5.40%	5.40%	5.40%	5.40%	5.40%
	T=2	6.16%	6.18%	6.21%	6.24%	6.28%
	T=3	6.35%	6.39%	6.47%	6.56%	6.65%
	T=4	6.95%	7.01%	7.09%	7.18%	7.27%
	T=5	7.43%	7.49%	7.57%	7.65%	7.74%
	T=7	7.75%	7.81%	7.90%	7.98%	8.06%
	T=10	8.04%	8.10%	8.18%	8.26%	8.34%
σ2 = 0.9	T=1	6.02%	6.02%	6.02%	6.02%	6.02%
	T=2	6.82%	6.83%	6.85%	6.88%	6.91%
	T=3	7.08%	7.11%	7.17%	7.24%	7.31%
	T=4	7.66%	7.70%	7.76%	7.83%	7.90%
	T=5	8.10%	8.14%	8.20%	8.27%	8.33%
	T=7	8.40%	8.44%	8.50%	8.57%	8.63%
	T=10	8.65%	8.70%	8.76%	8.82%	8.88%

Table 2 – Default probabilities for varying levels of sigma$_1$ and sigma$_2$

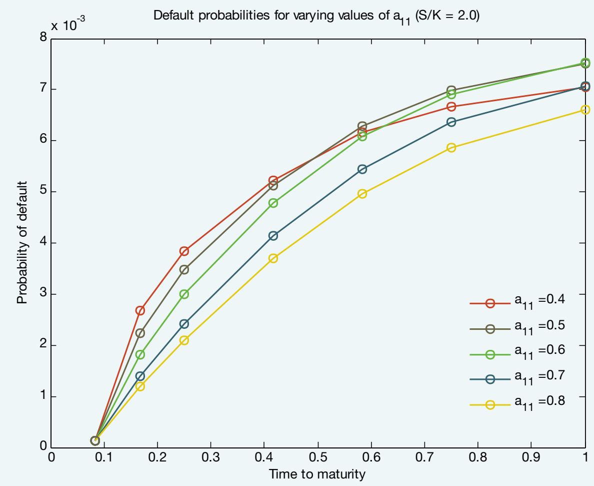

Figure 7 – Term structure of default probabilities for varying values of a_{11} for a term structure less than one year

Calibration of the structural model to CDS market data

As we are dealing with default probabilities of a sovereign, calibration of the structural model requires financial instruments that depend on these probabilities and aim to protect against a default event. In this regard CDSs are one of the most representative protection instruments [Duffie (1999)] since they protect the buyer against losses resulting from a credit event or default. CDSs are generally issued for a full range of sovereign bond issues, so that most countries' sovereign debt can be insured with CDSs because the contract obliges the issuer of the CDS to buy the defaulted bond at a predetermined price. The more likely that the reference entity will experience a credit event, the more valuable the contract will be. Instead of pricing CDSs according to a model, we can try to infer the model parameters by performing a reverse engineering procedure. At first we extract PDs from CDS market prices using an iterative process known as bootstrapping. The process starts by taking the shortest maturity contract, typically one year, and using it to extract the one-year survival probability and so forth, following a procedure similar to the one that allows us to bootstrap discount factors from swap rates. The bootstrapping method is illustrated in detail in Hull and White (2000) and O'Kane and Turnbull (2003).

Once we have extracted the default probabilities from quotes on CDS spreads for different maturities (1, 2, 3, 4, 5, 7, 10 years), the model calibration consists of inferring the model parameters, iteratively adjusted to best match the market observed default probabilities using non-linear least-squared-error minimization such as:

$$\left(par = \sigma_1, \sigma_2, a_{11}, a_{11}, X_0, \frac{S}{K}\right) = \min_{par} \sum_{j=1}^{T} \left(PD\left(t = 0, T_j\right)^{par} - PD(t = 0, T_j)^*\right)^2 \quad (8)$$

where $PD(t = 0,T_j)^*$ is the default probability obtained by bootstrapping the CDS quotes with maturity T_j observed at t = 0. Then we can use the calibrated model to infer the value of the sovereign assets that are otherwise unobservable. We now clarify this approach versus the classical use of the Merton model, such as the one applied by Gray et al (2007). They extend the Merton's contingent claim approach to the macro level to include a sovereign balance sheet analysis. This is done in the next section.

Applying a sovereign contingent claims approach

Random fluctuations in the market prices of an entity's assets and liabilities together with changes in financial inflows and outflows constitute balance sheet risk. If the total value of the assets falls below the level of promised payment on debt, distress and/or default occurs. The value of the risky debt is calculated as a default-free value of debt less an implicit put option on the underlying assets with the strike equal to the promised payments. Equity is modeled as an implicit call option with the same underlying asset and strike. The following balance sheet identity ensues:

Asset = Equity + Liability
= Implicit Call Option + Default-Free Debt – Implicit Put Option

The assets of a sovereign for the purpose of this approach are comprised of foreign reserves, net fiscal assets, and other assets minus entities too important to fail. Liabilities are defined as foreign-currency denominated debt plus a local-currency liability comprised of local-currency debt and base money. Sovereign default arises when sovereign assets cannot sufficiently cover the promised payment on foreign currency debt. The default barrier is therefore defined as the present value of these payments. While the liabilities are known with a fair degree of certainty over any given time horizon, the sovereign assets are more uncertain as assets may change for a large number of reasons. Three factors therefore drive default: the sovereign asset value, the volatility of the assets, and the default barrier. The default barrier may be defined as a KMV-like measure (short-term debt plus one-half long-term debt plus interest payments up to a certain time) or "senior" foreign-currency denominated debt [Crouhy et al. (2000]. This research adopts the latter definition. Seniority of debt is inferred from examining the behavior of policymakers during stress periods. Much effort is concentrated on remaining current on foreign-currency debt. These efforts make such debt more senior to "junior claims" on sovereign local-currency denominated debt.

When a lender makes a loan to a sovereign, an implicit guarantee of that loan equal to the expected loss of default is created. The action of the lender consists of pure default-free lending and bearing a risk of default by the sovereign. Risky debt can be viewed as a contingent claim on the (stochastic) sovereign assets. The foreign-currency debt can therefore be modeled as default-free value of debt minus an implicit put option. Local-currency liabilities are modeled as an implicit call option since such liability demonstrates "equity-like features" on a sovereign balance sheet. Excessive issue of both the money base and local-currency liabilities have a similar effect on inflation and price changes as the excessive issuing of corporate shares dilute shareholders' claims. The local-currency multiplied by the exchange rate is considered a market cap of the sovereign in the international market.

The main challenge is deriving an estimate for the market value and volatility of sovereign assets. Because these are not directly observable, the CCA approach relies on the relationship between balance sheet entries to extract an implied estimate of sovereign assets by a calibration procedure. The value of the local-currency liability in foreign currency terms (LCL) is a call option of the sovereign's assets (A) with the strike price as the default barrier (B_f) defined as foreign-currency denominated debt. The standard approach requires two equations: the first defines LCL as a call option on the asset value

$$LCL = AN(d1) - B_f\, e^{-r_f T}\, N(d2) \quad (9)$$

$$d_1 = \frac{ln\frac{A}{B_f} + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \quad (10)$$

The second equation defines the volatility of the LCL through

$$LCL^*\sigma_{LCL} = A\sigma_A\, N(d1) \quad (11)$$

while σ_A is the volatility of the sovereign's assets.

Equation (9) and (11) are typically used to calculate the unknown and unobservable sovereign asset value and asset volatility. The calibrated parameters can be used to obtain sovereign risk measures such as distance-to-default and probability of default and spreads on debt.

The important benefit is that the model provides a "fair value" estimate of debt and CDSs using balance sheet inputs and parameters. Its wide spectrum of use can help central banks analyze and manage the financial risks of the economy, showing the sensitivity of the enterprise's assets and liabilities to sudden external shocks [Gray et al. (2007)]. On the other hand, the estimates are strongly affected by the quality of the inputs.

Here we follow a different procedure. Instead of pricing CDSs or evaluating the value of balance sheet claims according to a CCA model, we use observed market data, filter it through the regime-switching model and we try to infer balance sheet information by performing a reverse engineering procedure. Here we use the calibrated parameters of the regime switching model to bootstrapped PDs to estimate the local-currency liabilities in foreign currency terms (LCL) as a call option on a sovereign's assets (A) with the strike price as the default barrier (B_f). An estimation of the sovereign unknown and unobserved asset value ($\hat{A}$) can be extracted from both the calibrated leverage parameter ($\frac{S}{K}$) and the observed distress barrier (B_f) such that

$$\hat{A} = \frac{S}{K} * B_f \tag{12}$$

The underlying risky asset (S) and the strike (K) in the regime-switching framework equates to the sovereign asset value (A) and threshold barrier (B_f). Similarly, the leverage parameter ($\frac{S}{K}$) equates to the ratio of the sovereign asset value (A) to the default threshold (A/B_f).

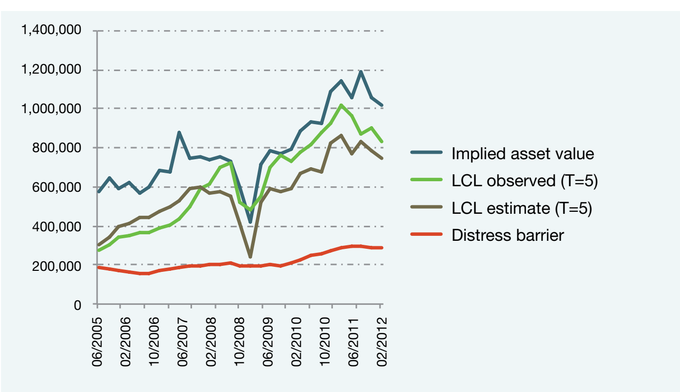

Figure 8 – Balance sheet estimates (in millions USD) – Brazil

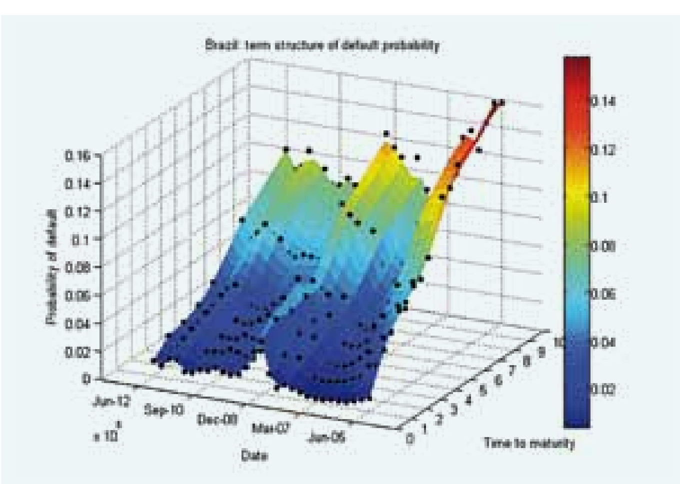

Figure 9 – Brazil: term structure of default probability – CDS bootstrap

The use of the Merton-type model requires many balance sheet inputs and parameters which are not always clearly observed and can sometimes be inaccurate or difficult to obtain. By reverse engineering the valuation of a sovereign's asset value, the model requires substantially less market information and adjusts for any structural breaks in the model in an attempt to improve the fair value estimates of a sovereign's balance sheet. The CCA approach shows the sensitivity of an enterprise's assets and liabilities to sudden external shocks whereas we attempt to adjust for these shocks through the regime-switching nature inherent in the volatility parameter. This approach could help one understand the movement of an economy from one state to another. It also allows us to appreciate the implied value of a sovereign's assets relative to existing debt that is observable and could signal a looming credit event. As an illustrative example, let us consider an application to Brazil.

Figure 8 illustrates a quarterly time series of the sovereign balance sheet components according to the model for an estimation period which extends post the financial crisis from June 2005 to June 2012 and a T=5 year maturity. The growing economic conditions in Brazil are evident by the rise in the indicators. The local-currency liabilities estimate overstates the observed value for periods prior to March 2008, proceeding to understate the observed value thereafter. The distress barrier remains well below the implied asset value aside from March 2009, where a sharp drawdown is experienced. The financial fragility displayed here is intimately related to the probability of default which is demonstrated in term structure of PDs around this time period in Figure 9. In the follow up paper a more extensive analysis will be performed on several countries.

Conclusion

Merton-style structural framework provides a very appealing feature that links credit risk to underlying structural variables via an endogenous description of credit defaults together with an intuitive economic interpretation. However, standard approaches to credit default probability estimation have certain drawbacks related to the undesirable property of underestimating the default probability, mainly over short-term periods. This research offers an attractive combination of possibly reducing the underestimation inherent in most standard structural models while establishing a link between the credit market and a sovereign's balance sheet in an attempt to understand if credit markets convey useful information to predict sovereign asset values. The crucial advantage of the methodology is the application of a regime-switching framework, which allows for structural shifts in the model, and substantially improves default risk estimation. Moreover, this methodology can be tractably extended to a CCA in the case of a sovereign, thereby obtaining a link between the credit market and predictions of a sovereign's balance sheet fundamentals.

We perform a sensitivity analysis for the term structure of the default probabilities under a Markov regime-switching framework when the model parameters vary. We consider a two-state Markov chain which distinguishes a strong economy from a weak economy or a normal market from one experiencing an economic crisis. The regime-switching model allows for switching of the economy between "good" and "bad" states. Therefore the higher risk of default is reflected in the higher default probabilities, illustrating the effects of the economic conditions captured by the Markov regime-switching principles. This is a possible way to explain and improve the underestimation of default probabilities associated with the Merton structural model [Erlwein et al. (2008]. On investigation, when you vary the asset value volatility, it becomes apparent that the default probabilities are sensitive to this parameter, with default probabilities increasing as volatility increases, irrespective of the starting state of the regime-switching process. Going forward, the research focus does not attempt to predict the starting state of the economy but rather the persistence in a state defined by the strength of the transition probabilities and the economic conditions as defined by the volatility.

The modeling apparatus can be applied to sovereign risk. This will be investigated in much more detail in a follow up paper where CDS quotes are used to calibrate the regime-switching model and then this is used to estimate sovereign assets, in both developed and emerging markets.

References

- Baum, L., and T. Petrie, 1966, "Statistical Inference for Probabilistic Functions of Finite State Markov Chains," The Annals of Mathematical Statistics, 6(37): 1554–1563
- Bhattacharya, S., and A. Gupta, 1980, "Occupation Times for Two-State Markov Chains," Journal of Discrete Applied Mathematics, 2(3): 249–250
- Black, F., and J. Cox, 1976, "Valuing Corporate Securities: Some Effects of the Bond Indenture Provisions," The Journal of Finance, 81: 351-367
- Black, F., and M. Scholes, 1973, "Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81: 637-654
- Boral, A., L. Carty, and E. Falkenstein, 2000, RiskCalc Private Model: Moody's Default Model for Private Firms, Moody's Investors Service
- Brigo, D., and M. Tarenghi, 2005, "Credit Default Swap Calibration and Counterparty Risk Valuation with a Scenario Credit Default Swaps I: No Counterparty Default Risk," Journal of Derivatives, 8: 29-40
- Brigo, D., M. Morini and M. Tarenghi, 2009, "Credit Calibration with Structural Models: The Lehman Case and Equity Swaps Under Counterparty Risk," http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1530742
- Crouhy, M., D. Galai and R. Mark, 2000, "A Comparative Analysis of Current Credit Risk Models," Journal of Banking Finance, 24: 59-117
- Delatte, A., M. Gex, and A. Lopez-Villavicencio, 2012, "Has the CDS Market Influenced the borrowing cost of European countries during the sovereign crisis?" Journal of International Money and Finance, 31: 81-497
- Duffie, D., 1999, "Credit Swap Valuation," Financial Analysts Journal, 55: 73-89
- Duffie, D., 2010, CDS on Government Debt. Washington: Hearing before the US House of Representatives.
- Duffie, D., L. Saita, and K. Wang, 2007, "Multi-Period Corporate Default Prediction with Stochastic Covariates," Journal of Financial Economics, 83(3): 635-665
- Erlwein, C., R.S. Mamon, and T.K. Sui, 2008, "The Pricing of Credit Default Swaps under a Markov-Modulated Merton's Structural Model," Northern American Actuarial Journal, 12(1): 19-46
- Gapen, M., D. Gray, C. Lim, and Y. Xiao, Y, 2008, "Measuring and Analyzing Sovereign Risk," IMF Staff Papers, 55(1): 109-148
- Giesecke, K., F. A. Longstaff, and S. Schefer, 2011, "Corporate bond default risk: A 150-year perspective," Journal of Financial Economics, 102: 233-250
- Gray, D., M. Merton, and Z. Bodie, 2007, "Contingent Claims Approach to Measuring and Managing Sovereign Credit Risk," Journal of Investment Management, 5: 5-28
- Hamilton, J., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," Econometrica, 57:357-384
- Hull, J., and A. White, 2000, "Valuing Credit Default Swaps I: No Counterparty Default Risk," Journal of Derivatives, 8: 29-40
- Ishijima, H., and T. Kihara, 2005, Option Pricing with Hidden Markov Models. Institute of Finance, Waseda University
- Kupiec, P. H., 2002, Internal Models-Based Capital Regulation and Bank RiskTaking Incentives. IMF Working Paper No. 02/125
- Leland, H., 2004, "Predictions of Default Probabilities in Structural Models of Debt," Journal of Investment Management, 2: 1-28
- Liew, C. C., and T. K. Siu, 2010, "A hidden Markov regime-switching model for option valuation," Insurance: Mathematics and Economics, 47: 374-384
- Longstaff, F., and E. Schwartz, 1995, "Valuing credit derivatives," The Journal of Fixed Income, 5: 6-12
- Merton, R., 1973, "On the pricing of corporate debt: risk structure of interest rates," Journal of Finance, 29: 449-470
- Merton, R., 1977, "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory," Journal of Banking and Finance, 1: 3-11
- O'Kane, D., and S. Turnbull, 2003, Valuation of Credit Default Swaps. Lehman Brothers, Fixed Income Quantitative Credit Research
- Sundaresan, S. M., 2000, "Continuous-time methods in finance: A review and an assessment," The Journal of Finance, 55(4): 1569-1622
- Tarashev, N., 2005, An Empirical Evaluation of Structural Credit Risk Models. Working Paper, Bank of International Settlements
- Timmermann, A., 2000, "Moments of Markov Switching Models," Journal of Econometrics, 96, 75-111

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